

Nonfactorizable Corrections to Hadronic Weak Decays of Heavy Mesons*

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ABSTRACT

Status of nonfactorizable effects in exclusive hadronic weak decays of D and B mesons is reviewed.

1. Introduction

It is customary to make the factorization approximation to describe the hadronic weak decays of mesons; that is, the meson decay amplitude is dominated by the factorizable terms provided that final-state interactions and nonspectator contributions are negligible. The hadronic matrix elements of the factorizable amplitude is factorized into the product of two matrix elements of single currents, governed by decay constants and form factors. Consider a generic two-body decay of a meson $M \rightarrow M_1 + M_2$. The factorizable parts of the decay amplitude can be classified into three different categories¹:

$$\begin{aligned} \text{class I (external W emission) : } & a_1 \langle M_1 | (\bar{q}_1 q_2) | 0 \rangle \langle M_2 | (\bar{q}_3 q_4) | M \rangle, \\ \text{class II (internal W emission) : } & a_2 \langle M_2 | (\bar{q}_3 q_2) | 0 \rangle \langle M_1 | (\bar{q}_1 q_4) | M \rangle, \end{aligned} \quad (1)$$

and the third class involving decays in which a_1 and a_2 amplitudes interfere. Meson M_1 in class I decays is generated from the charge current $(\bar{q}_1 q_2) \equiv \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2$, while M_2 in the class II transition comes from the neutral current $(\bar{q}_3 q_2)$. For a given parent meson M , the parameters a_1 and a_2 are universal and channel independent. For the QCD-corrected effective weak Hamiltonian

$$\mathcal{H}_{\text{eff}} \propto c_1 O_1 + c_2 O_2 = c_1 (\bar{q}_1 q_2) (\bar{q}_3 q_4) + c_2 (\bar{q}_1 q_4) (\bar{q}_3 q_2), \quad (2)$$

the parameters a_1 , a_2 are related to the Wilson coefficient functions c_1 and c_2 via

$$a_1 = c_1 + c_2/N_c, \quad a_2 = c_2 + c_1/N_c \quad (3)$$

in the standard factorization approach, where the term proportional to $1/N_c$ arises from the Fierz transformation.

However, it is known that this factorization approach fails to describe class II charmed decay modes, e.g., $D^0 \rightarrow \bar{K}^0 \pi^0$, $D^+ \rightarrow \phi \pi^+$, \dots , etc.² For example, the ratio

*To appear in the proceedings of *The International Symposium on Particle Theory and Phenomenology*, Iowa State University, May 22-24, 1995.

$\Gamma(D^0 \rightarrow \bar{K}^0 \pi^0)/\Gamma(D^0 \rightarrow K^- \pi^+)$ is predicted to be ~ 0.02 , whereas experimentally it is measured to be 0.51 ± 0.07 .³ It was realized by several groups⁴ that the discrepancy between theory and experiment is greatly improved if Fierz transformed terms in (3) are dropped. It has been argued that⁵ this empirical observation is justified in the so-called large- N_c approach in which a rule of discarding subleading $1/N_c$ terms can be formulated. There are several important implications of such a approach: (i) The factorization hypothesis for nonleptonic meson decays is justified in the limit of $N_c \rightarrow \infty$ since nonfactorizable contributions are suppressed relative to the factorizable ones by at least factors of N_c . (ii) Color suppression in the class II transitions is no longer operative in charm decay as $a_2 = c_2(m_c) \approx -0.52$ and $a_1 = c_1(m_c) \approx 1.26$. The sizable destructive interference partially accounts for the longer lifetime of D^+ relative to D^0 . (iii) Contrary to the meson case, the factorization approximation is not applicable to hadronic baryon decays. The nonfactorizable W -exchange contributions, which manifest as pole diagrams at the hadronic level, are no longer helicity and color suppressed; color suppression is compensated by a combinatorial factor of order N_c stemming from the fact that the baryon contains N_c quarks in the large- N_c limit.

Though the new $1/N_c$ factorization improves substantially over the standard factorization for charm decay, it cannot be a universal approach for describing the nonleptonic weak decays of mesons. First, a theory by itself should be able to specify the regime where it is applicable. However, there is no kinematic region where the $1/N_c$ expansion is guaranteed to be valid. Second, it fails to explain the constructive interference recently observed in charged B decays⁶: $B^- \rightarrow D^0 \pi^-$, $D^0 \rho^-$, $D^{*0} \pi^-$. Therefore, whether or not the large- N_c picture works is at best case by case dependent. If it operates, there must exist some dynamical reason for the suppression of the $1/N_c$ terms. This implies that nonfactorizable terms should play an essential role and it is our purpose to examine such effects.

2. Nonfactorizable effects in $D \rightarrow PP$, VP decays

Considering the class-I decay $D_s^+ \rightarrow \phi \pi^+$ as an example and using the Fierz identity

$$O_{1,2} = \frac{1}{N_c} O_{2,1} + \tilde{O}_{2,1}, \quad (4)$$

where $\tilde{O}_1 = \frac{1}{2} \bar{q}_1 \gamma_\mu (1 - \gamma_5) \lambda^a q_2 \bar{q}_3 \gamma^\mu (1 - \gamma_5) \lambda^a q_4$, we find [†]

$$\begin{aligned} \langle \phi \pi^+ | \mathcal{H}_W | D_s^+ \rangle &\propto (c_1 + \frac{c_2}{N_c} + c_2 \chi_1) \langle \pi^+ | (\bar{u}d) | 0 \rangle \langle \phi | (\bar{s}c) | D_s^+ \rangle + c_1 \langle \phi \pi^+ | \tilde{O}_2 | D_s^+ \rangle \\ &+ (c_1 + \frac{c_2}{N_c}) \langle \phi \pi^+ | O_1 | D_s^+ \rangle_{nf} + (c_2 + \frac{c_1}{N_c}) \langle \phi \pi^+ | O_2 | D_s^+ \rangle_{nf}, \end{aligned} \quad (5)$$

[†]Recall that in the standard picture the hadronic matrix element of the operator O is evaluated by considering the contributions of the operator itself and its Fierz transformation.

where $\chi_1 = \langle \phi\pi^+ | \tilde{O}_1 | D_s^+ \rangle / \langle \phi\pi^+ | O_1 | D_s^+ \rangle_f$ and the subscript nf denotes nonfactorizable corrections to the matrix elements of $O_{1,2}$. The nonfactorizable terms $\langle \phi\pi^+ | O_{1,2} | D_s^+ \rangle_{nf}$ and $\langle \phi\pi^+ | \tilde{O}_2 | D_s^+ \rangle$ in (5) are usually ignored in the literature.

To proceed, we will assume that the nonfactorizable contributions are dominated by the color-octet current \tilde{O}_1 . Consequently, $\langle \phi\pi^+ | \mathcal{H}_W | D_s^+ \rangle \propto a_1^{\text{eff}} \langle \pi^+ | (\bar{u}d) | 0 \rangle \langle \phi | (\bar{s}c) | D_s^+ \rangle$ with

$$a_1^{\text{eff}} = c_1 + c_2 \left(\frac{1}{N_c} + \chi_1 \right). \quad (6)$$

Likewise, nonfactorizable terms for the class II decay modes amount to a redefinition of a_2 :

$$a_2^{\text{eff}} = c_2 + c_1 \left(\frac{1}{N_c} + \chi_2 \right). \quad (7)$$

(For convenience, we will drop the superscript “eff” henceforth.) The key point is that the amplitudes of $D, B \rightarrow PP, VP$ are governed by a single form factor so that nonfactorizable contributions due to final-state soft gluon effects can be lumped into the effective parameters a_1 and a_2 . Though we do not know how to perform first-principles calculations of $\chi_{1,2}$, we do expect that⁷

$$|\chi(B \rightarrow PP)| < |\chi(D \rightarrow PP)| \lesssim |\chi(D \rightarrow VP)|, \quad (8)$$

as soft gluon effects become stronger when the relative momentum of the final-state particles becomes smaller, allowing more time for significant final-state interactions (FSI).

Because of the presence of FSI and the nonspectator contributions, it is generally not possible to extract the nonfactorization parameters $\chi_{1,2}$ except for a very few channels. Therefore, in order to determine a_1 and especially a_2 we should focus on the exotic channels and the decay modes with one single isospin component where nonspectator contributions are absent and FSI are presumably negligible. From data we find that⁸

$$\begin{aligned} \chi_2(D \rightarrow \bar{K}\pi) &\simeq -0.36, & \chi_2(D \rightarrow \bar{K}^*\pi) &\simeq -0.61, \\ \chi_2(D^+ \rightarrow \phi\pi^+) &\simeq -0.44, & \chi_1(D_s^+ \rightarrow \phi\pi^+) &\simeq -0.60, \end{aligned} \quad (9)$$

where we have assumed $\chi_1 \simeq \chi_2$ for $D \rightarrow \bar{K}^{(*)}\pi$ decays. Note that, as pointed out in Ref.[9], the solutions for χ are not uniquely determined. For example, another possible solution for $\chi_2(D \rightarrow \bar{K}\pi)$ is -1.18 . To remove the ambiguities, we have assumed that nonfactorizable corrections are small compared to the factorizable ones. We see from (9) that in general $\chi_{1,2}$ and hence $a_{1,2}$ are not universal and they are channel dependent and satisfy the relation $|\chi(D \rightarrow PP)| < |\chi(D \rightarrow VP)|$ as expected. We also see that since $\chi_2(D \rightarrow \bar{K}\pi)$ is close to $-\frac{1}{3}$, it is evident that a large cancellation between $1/N_c$ and $\chi_2(D \rightarrow \bar{K}\pi)$ occurs. This is the dynamic reason why the large- N_c approach

operates well for $D \rightarrow \bar{K}\pi$ decay. However, this is no longer the case for $D \rightarrow VP$ decays; the predicted decay rates for $D \rightarrow VP$ in the large- N_c approach in general disagree with data.⁸ Therefore, we are led to conclude that the leading $1/N_c$ expansion cannot be a universal approach for the nonleptonic weak decays of the meson. However, the fact that substantial nonfactorizable effects which contribute destructively with the subleading $1/N_c$ factorizable contributions are required to accommodate the data of charm decay means that, as far as charm decays are concerned, the large- N_c approach greatly improves the naive factorization method in which $\chi_{1,2} = 0$; the former approach amounts to having universal nonfactorizable terms $\chi_{1,2} = -1/N_c$.

3. Nonfactorizable effects in $B \rightarrow PP$, VP decays

If the large- N_c picture is a universal approach for hadronic weak decays of mesons, one will expect that $a_2(B) \simeq c_2(m_B) \approx -0.26$. However, CLEO data⁶ clearly indicate a constructive interference in charged B decays $B^- \rightarrow D^0\pi^-$, $D^0\rho^-$, $D^{*0}\pi^-$ and hence a positive a_2 . This is a very stunning observation since it has been widely believed by most practitioners in this field that the $1/N_c$ expansion applies equally well to the weak decays of the B meson.

Using the heavy-flavor-symmetry approach for heavy-light form factors and assuming a monopole extrapolation for F_1 , A_0 , A_1 , a dipole behavior for A_2 , V , and an approximately constant F_0 , as suggested by QCD sum-rule calculations and some theoretical arguments,¹⁰ we found from CLEO data that the variation of $a_{1,2}$ from $B \rightarrow D\pi$ to $D^*\pi$ and $D\rho$ decays is negligible and the combined average is¹⁰ ‡

$$a_1[B \rightarrow D^{(*)}\pi(\rho)] = 1.01 \pm 0.06, \quad a_2[B \rightarrow D^{(*)}\pi(\rho)] = 0.23 \pm 0.06, \quad (10)$$

where we have neglected FSI and nonspectator effects, an assumption which is probably justified in B decays. It follows that

$$\chi_1[B \rightarrow D^{(*)}\pi(\rho)] \simeq 0.05, \quad \chi_2[B \rightarrow D^{(*)}\pi(\rho)] \simeq 0.11. \quad (11)$$

Since $|c_2| \ll |c_1|$, it is clear that the determination of χ_1 is far more uncertain than χ_2 . Evidently, soft gluon effects are less significant in B decays, as what expected [see (8)]. For $B \rightarrow \psi K$ decays, we found¹⁰

$$|a_2(B \rightarrow \psi K)| = 0.225 \pm 0.016. \quad (12)$$

We have argued that its sign is positive since $\chi_2(B \rightarrow \psi K)$ should not deviate too much from $\chi_2(B \rightarrow D\pi)$. It has been advocated by Soares⁹ that an analysis of the long-distance contribution of $B \rightarrow \psi K$ to the decay $B \rightarrow K\ell^+\ell^-$ can be used to remove the sign ambiguity of a_2 .

[‡]The number $a_2/a_1 = 0.23 \pm 0.11$ given in the CLEO paper⁶ is obtained by a global least squares fit of the modified Bauer-Stech-Wirbel model¹¹ to the CLEO data of $B \rightarrow D^{(*)}\pi(\rho)$. (An individual fit of the same model to the data gives rise to the average $a_2/a_1 = 0.33 \pm 0.08$.) Our result $a_2/a_1 = 0.22 \pm 0.06$ thus improves the previous error analysis by a factor of 2.

Thus far the nonfactorizable effect is discussed at a purely phenomenological level. It is very important to have a theoretical estimate of such effects even approximately. So far all existing theoretical calculations rely on the QCD sum rule. The first pioneering work is due to Blok and Shifman¹² who calculated soft gluon contributions and found that $1/N_c$ factorizable terms and the soft gluon effect χ almost compensate in all $D \rightarrow PP$ decays and in some decay modes of $D \rightarrow VP$. They have applied the same approach to the class I decay $\bar{B}^0 \rightarrow D^+\pi^-$ and obtained¹³ $\chi_1(\bar{B}^0 \rightarrow D^+\pi^-) \sim -0.5$. Working in the framework of the light-cone QCD sum rule, Rückl and his collaborators¹⁴ found a large cancellation between the Fierz $1/N_c$ term and the nonfactorizable contribution χ_2 . Most recently, Halperin¹⁵ extended the same calculation to the class II decay $\bar{B}^0 \rightarrow D^0\pi^0$ and found $\chi_2(\bar{B}^0 \rightarrow D^0\pi^0) \sim -0.35$ and hence a negative $a_2(B \rightarrow D\pi)$, which is in contradiction with experiment. It appears that all present QCD sum-rule calculations tend to imply that the rule of discarding $1/N_c$ terms seems to hold in class-I and class-II decays of the B meson. It is thus a great challenge to the theorists to understand the origin of disagreement between theory and experiment for the parameter $a_2(B \rightarrow D\pi)$. This tantalizing issue should be clarified and resolved in the near future. At present, lattice calculations of soft gluon effects are already available for $D \rightarrow \bar{K}\pi$ decay. An extension of such a computation to class II decay modes of the B meson is urged.

4. Nonfactorizable effects in $B, D \rightarrow VV$ decays

The study of nonfactorizable effects in $M \rightarrow VV$ decay is more complicated as its general amplitude consists of three independent Lorentz scalars:

$$A[M(p) \rightarrow V_1(\varepsilon_1, p_1)V_2(\varepsilon_2, p_2)] \propto \varepsilon_\mu^*(\lambda_1)\varepsilon_\nu^*(\lambda_2)(\hat{A}_1 g^{\mu\nu} + \hat{A}_2 p^\mu p^\nu + i\hat{V}\epsilon^{\mu\nu\alpha\beta}p_{1\alpha}p_{2\beta}), \quad (13)$$

where $\hat{A}_1, \hat{A}_2, \hat{V}$ are related to the form factors A_1, A_2 and V respectively. Since *a priori* there is no reason to expect that nonfactorizable terms weight in the same way to S -, P - and D -waves, namely $A_1^{nf}/A_1 = A_2^{nf}/A_2 = V^{nf}/V$, we thus cannot define χ_1 and χ_2 . Consequently, *it is in general not possible to define an effective a_1 or a_2 for $M \rightarrow VV$ decays once nonfactorizable effects are taken into account.*¹⁶

It was pointed out recently that there are two experimental data, namely the production ratio $R \equiv \Gamma(B \rightarrow \psi K^*)/\Gamma(B \rightarrow \psi K)$ and the fraction of longitudinal polarization Γ_L/Γ in $B \rightarrow \psi K^*$, which cannot be accounted for simultaneously by all commonly used models within the framework of factorization.¹⁷ The experimental results are

$$R = 1.74 \pm 0.39, \quad \frac{\Gamma_L}{\Gamma} = 0.78 \pm 0.07, \quad (14)$$

where the latter is the combined average of the measurements by ARGUS, CDF¹⁸ and CLEO.⁶ Irrespective of the production ratio R , all the existing models fail to produce a large longitudinal polarization fraction.¹⁷ This strongly implies that the puzzle with Γ_L/Γ can be resolved only by appealing to nonfactorizable effects. However, if the relation $A_1^{nf}/A_1 = A_2^{nf}/A_2 = V^{nf}/V$ holds, then an effective a_2 can be defined for

$B \rightarrow \psi K^*$ and the prediction of Γ_L/Γ will be the same as that in the factorization approach as the polarization fraction is independent of a_2 . As a result, *nonfactorizable terms should contribute differently to S-, P- and D-wave amplitudes if we wish to explain the observed Γ_L/Γ .*

From data of $B \rightarrow \psi K^*$ and $D \rightarrow \bar{K}^* \rho$ decays, we found that the nonfactorizable terms in $M \rightarrow VV$ decay in general satisfy the relation⁸

$$\frac{A_1^{nf}}{A_1} > \frac{A_2^{nf}}{A_2}, \quad \frac{V^{nf}}{V}. \quad (15)$$

The difference between $B \rightarrow VV$ and $D \rightarrow VV$ decays stems from the fact that A_1^{nf} is positive in the former, while it is negative in charm decay. Since the magnitude of A_2^{nf}/A_2 is larger than that of A_1^{nf}/A_1 in $D \rightarrow \bar{K}^* \rho$ decay, it is evident that the assumption of S-wave dominance for nonfactorizable terms fails in charm decay. We thus urge experimentalists to measure the polarized decay rates in the color- and Cabibbo-suppressed decay mode $D^+ \rightarrow \phi \rho^+$ decay to gain insight in the nonfactorizable effects in $D \rightarrow VV$ decay.

Acknowledgment This work was supported in part by the National Science Council of ROC under Contract No. NSC84-2112-M-001-014.

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